The Underground economy in Spain: A Bayesian DSGE approach

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Abstract

This paper studies the Spanish Underground economy by estimating its size and trend with a Bayesian Dynamic Stochastic General Equilibrium (DSGE) approach for the period 1980:Q1-2015:Q4. I found that the Spanish shadow economy has a large variation, ranging from a minimum of 16% to a maximum of more than 35%, over the whole period due to its labor dependency. Particularly, it suffers a strong decrease at the beginning of the financial crisis, followed by a continuous growth until year 2016. Regarding to the fiscal policy analysis, the estimated results suggest that Spain can increase its fiscal revenues by raising both corporate tax rate and household income tax rate, but with an important negative effect over social security contributions. Finally, a sensitivity analysis of the model estimation indicates that the potential gains of increasing the cost of supplying labor to the irregular production overcome the increase in fiscal revenues from a tax rise.

1 Introduction

The underground economy is an important issue for a large number of countries around the world and, as emphasized by Schneider (2000) it was taking a major influence on the world economy. This is due to its harmful effects for those countries with a high level of irregular economy, which can be summarized in the followings points as it shows in Arrazola et al. (2011): (1) the shadow economy generates inequality problems, (2) raises problems of efficiency, (3) produces a decrease in tax revenue, (4) distorts business competition and (5) makes data less reliable. Obviously, consequences 1-4 generate a decrease in social welfare, but the fifth problem could be as important as the previous ones, not only due to its effects on economic research, but also due to its effects on economic and fiscal policies [1].

In this regard, it is important to list the causes of the underground economy, i.e. those factors that determine its size and trend. This is a significant issue because it can help us to better understand the behavior and influence of the

[1] The resulting optimal policy could be very different if we take into account the existence of an irregular production sector or not. This is highlighted in Orsi et al. (2013), they show that the resulting optimal fiscal policy for the Italian economy changes taking into account the shadow economy.

Table 1: Underground Economy

	Median (%)	Minimum (%)	Maximum (%)	Billions of euros
Germany	15.9	13.3	17.7	5557
United Kingdom	13.1	11.7	15.8	3471
France	15.8	14.0	18.2	393
Italy	29.0	26.8	33.5	537
Spain	25.6	22.7	28.7	321
Netherlands	14.0	13.0	16.0	108
Switzerland	9.1	8.0	10.0	60
Sweden	19.2	16.7	24.5	99
Belgium	23.4	21.6	25.8	110
Poland	27.5	19.1	34.5	129

Table 1: Size of the Underground economy for the top ten economies in the European Union. Column 1-3 display the median, minimum and maximum value of the underground economy as percentage of total output. Column 4 presents the size of the informal economy measured in billions of euros.

irregular production in an economy. There are six main causes identified by Schneider and Buehn (2013) that trigger movements in the underground economy: tax burden, quality of the institutions, regulation, public sector services, tax moral and deterrence. Taking into account these aspects, in this paper I build a DSGE model for the Spanish economy that explicitly consider the existence of underground sector and tax evasion with the final goal of estimating their importance for the Spanish economy. Particularly, as a novelty in this type of model and following Anzoategui (2016), I use mixed frequency data by including annual information about the social security contributions in Spain and tax enforcement activities by the government, together with quarterly data of consumption, investment, wages and fiscal revenues. Social security contributions are included in order to control for the most important variable in explaining the existence of the underground sector whereas, I include the intensity of firm's inspections to cover two causes, regulation and deterrence. Secondly, public sector services are somehow reflected in the model by the parameter that symbolizes the utility cost of working in the underground sector. For simplicity, I omit parameters that reflect the quality of the institutions due to the lack of data and the subjective meaning of this cause [2].

The underground economy in Spain is a particularly important issue, due to its size and historical persistence. In the European Union, Spain is among the economies with the highest levels of underground economy measured as a percentage of the GDP (Medina and Schneider (2017)), as it is shown in Table 1, which provides estimates of the underground economy for the top ten economies in the European Union. More precisely, table 1 displays the median,

[2]It is possible and relatively simple to consider this issue by introducing a parameter that reflects public corruption as it is shown in Pappa et al. (2014).

minimum and maximum value of the informal economy for the period 1991-2015. As we can see, Spain is the third country with higher shadow economy in the top ten European economies, preceded only by Italy and Poland. In terms of GDP the irregular economy represents 320 billions of euros, a quantity that exceeds Spanish total fiscal revenues in the year 2017 (292 billions of euros). These results highlight the importance of the underground economy in Spain and motivate the analysis of the variables that affect its trend and variation with the final goal of choosing proper fiscal and economic policies. Therefore, this study tries to give an estimation of the irregular economy and its dynamic, along with the main variables that explain its behavior for the period 1980:Q1-2015:Q4, by estimating a two sector Dinamic Stochastic General Equilibrium (DSGE) model with a bayesian approach.

Regarding the Spanish irregular economy, there are several methods that estimates its size for Spain, the most important ones are presented by Jiménez and Martínez-Pardo (2013), Pickhardt and Sardá (2015) and Medina and Schneider (2017) among others. All the available methods used for estimating the Spanish Underground economy are base in statistics approaches and the existent data, without taking into account any assumption about the rational behavior of agents. Therefore, it is not possible to apply a proper fiscal policy analysis that helps to reduce the size of the informal economy. This study tries to give an answer to this problem following the approach pursued in Orsi. et al (2013), where they estimate the Italian informal economy using a two sector DSGE model with three types of agents: households, firms and government. For Spain I use a similar model with different assumptions about the irregular production functional form and the input used in this sector, adding habit formation in the household utility and capital adjustment cost. Also, I execute a deep analysis of the effects of corporate tax rate and household income tax rate on social security fiscal revenues, due to its importance in maintaining social expenditures like pensions or unemployment benefits among others.

The motivation of this analysis comes from the fact that there is a slow recovery of the social security contributions as it is shown in Table 2. Particularly, first column presents the dates in which social security contributions and fiscal revenues from corporate taxation reach their maximum levels. Second and third columns show the minimum level of the variables and their levels in the fourth quarter of 2015, respectively. Finally, last column displays the average growth rate in the period 2013:Q1-2015:Q4 for both social security contributions and firms fiscal revenues. There is a larger variation on fiscal revenues from corporate tax rate than for social security contributions, but taking into account the actual level of both variables we can see a belated recovery of the second variable, which is 8 percentage points lower than in 2008:Q1. Also, Table 2 presents the average growth rate of both, social security contributions and corporate fiscal revenues, showing that social security fiscal revenues growth rate is much more lower than its homologous case.

Therefore, there is a late recovery of social security contributions that hurts the Spanish social system, which nowadays has an important deficit and needs from other resources to finance crucial social expenditures in terms of social

Table 2: Fiscal Revenues Dynamics

	Maximum Level=100%	Minimum Level (%)	2015:Q4 (%)	Average Growth Rate (2013:Q1-2015:Q4)
S. S. Contributions	2008:Q2	87.75	92	0.27
Firms Fiscal Revenues	2006:Q4	65.65	94.75	1.30

welfare. Consequently, it needs from a special control of its behavior and the indirect effects that comes from other fiscal policies.

Moreover, since fiscal policy affects the economics decisions of individuals and firms, it has an important impact on the size of the underground economy as it is shown by Orsi et al. (2013), Busato and Chiarini (2004,2013) among others. For this reason, I analyze the effects of fiscal policies on the dynamics of the estimated underground economy by using orthogonalized impulse response functions to shocks in the main variables, with the final goal of testing the model implications.

Concerning the available methods for estimating the underground economy, the most important problem comes from the fact that the underground economic activities are not observable by definition. This is the main cause that explains the existence of several and different approaches used to estimate the informal economy. As emphasized by Schneider (2014), Jiménez and Martínez-Pardo (2014) among others, we can distinguish between three main methods of estimation: direct approaches, indirect approaches and model approaches.

- Direct approaches: this estimation method is based on microeconomic approaches and tries to determine the size of the underground economy by using survey data, but it has several important problems like the type of questionnaires to be performed and the robustness of the individuals answer, i.e. not all individuals are willing to say that they are working in the underground sector. It is also important to note that this type of approach requires large samples to estimate the size of the shadow economy, which implies an important cost. No estimates are available for Spain using this method, due to the absence of data. However, it has been carried out for other countries like Denmark in the paper by Pedersen (2003).

- Indirect approaches: These methods are based on macroeconomic magnitudes and data of the market economy that are used as a proxies to estimate the size of the underground economy and its trend. As in the paper by Jiménez and Martínez-Pardo (2013), they try to estimate an unobservable variable by using the behavior of others variables that are observables and some condition about the relationship between the observable variables and the unobservable variables. Within this approach there are several method that has been used for estimating the Spanish Underground economy (see Arrazola et al. (2011), but the most important ones are the currency demand approach and the physical input method:

- The currency demand approach was presented by Cagan (1958) and it is based on the idea that the underground economy uses a legal payment method. Therefore, there exists a relationship between the currency demand and the underground economy. The main criticism to this method is that not all transactions in the underground economy are done with legal payment method (Schneider and Buehn (2013)).
- The physical input (electricity consumption) method uses inputs consumed in the production process to estimate the volume of shadow economy. Electricity consumption is the most common variable used to estimate the irregular production because it is pretty easy to measure its demand. However, it is obvious that not all the production processes need electricity as a significant input (Schneider and Buehn (2013)).

Finally, there is another type of estimation approach, which is based in econometrics models and statistical tools that are combined with the final goal of estimating the shadow economy as a latent variable. Therefore, since I'm using a model to infer the dynamic of the underground production, the bayesian estimation of the two sector DSGE model that I implement in this paper belong to this type of approach. This approach is called the model approach and among its estimation methods we can highlight the Multiple Indicators and Multiple Causes method (MIMIC) and the of DSGE model estimation (Kireenko and Nevzorova (2015)). First, regarding to the MIMIC method, it uses a set of observable variables that cause movements in the latent variable (underground economy) and a set of observable indicators that are influenced by the size of the underground economy. Also, this is one of the most used method for estimating the Spanish underground economy. However, as Helberger and Knepel (1988) and Vázquez et al. (2010) pointed out, it has some limitations: the instability of the coefficients when the sample size or the model specification change, it is difficult and costly to obtain data from the causal variables and the reliability of the variables and indicators used in the estimation process.

Secondly, the methods that use Dynamic Stochastic General Equilibrium models to estimate the informal economy are based, as pointed out by Kireenko and Nevzorova (2015), on the assumption of rational behavior of agents which solves maximization problems. More precisely, Orsi et al. (2013) apply this methodology for the Italian economy highlighting its potential capability in analyzing fiscal and economic policies. They study whether the actual corporate tax rate and household income tax rate are efficient in order to propose fiscal policies that are welfare improving. They found that the Italian fiscal system was using an inefficient tax rate for both, corporate taxes and household income taxes, and suggest that fiscal revenues may increase as a consequence of a decrease in taxes.

I apply a two sector DSGE model following Orsi et al. (2013), under some variations in the model specification, with the final goal of estimating the Spanish Underground economy and its dynamic. Also, I present a policy implication analysis regarding to the fiscal revenues of both, corporate taxation and household income taxation. I find that Spain can increase fiscal revenues by rising corporate and household income tax rate. However, these policies are very costly if we take into account the significant decrease of fiscal revenues that come from social security contributions due to its importance in terms of social welfare. This importance is due to the fact that social security contributions are used to finance public pensions, unemployment benefits and others social expenditures. On the other hand, I found that total fiscal revenues and social security contributions may increase by rising the cost of supplying labor to the underground sector without negative effects in other variables, which can be done by increasing social benefits of working in the regular sector. My results highlight the significance of taking into account indirect effects of fiscal policies in order to select the most efficient one.

This paper is organized as follows: Section 2 presents a detailed description of the model and the optimality conditions. Section 3 describes the bayesian estimation of the DSGE model and the data used. Also, it displays the prior distributions of the estimated parameters and their resulting posterior distributions. In Section 4 I carry on a robustness check of the estimated results by estimating a different length of data. The Dynamics followed by the underground production and its main components are described in Section 5, while the main policy implications are exhibited in Section 6. Finally, Section 7 summarizes the main conclusions of this study.

2 The model

I consider an extension of the DSGE model used by Orsi et al. (2013) which consist in three different types of agents: households, firms and government. In order to adapt the model for Spanish case, I have introduced some modifications with respect to the original framework together with different assumptions about the household's utility and the law of motion.

2.1 Firms

There is a continuum of homogeneous goods indexed by $i \in [0,1]$, each produced by a perfectly competitive producer. Firms use two different Cobb-Douglass production functions which differ in the type and the number of inputs used to produce the same type of output. [3] The regular production function combines regular labor h_t^m and physical capital k_t to produce regular output according to the following specification:

$$y_{i,t}^m = A_t \left(\Gamma_t h_{i,t}^m\right)^\alpha k_{i,t}^{1-\alpha} \tag{1}$$

[3] In contrast to the original model, I have avoid the existence of irregular capital in the economy with the final goal of adapt the model to the Spanish economy case. This assumption stems from the fact that the underground production is labor intensive and in most of the cases it uses regular capital in the production process.

where α is the regular labor elasticity and A_t is a purely transitory technological shock that affects to the regular production. The regular production is taxed at the corporate tax rate τ_t^c which follows an stochastic AR(1) process, but firms can decide whether to produce using legal inputs or illegal inputs. In the last case firms would use the irregular production function:

$$y_{i,t}^u = B_t \left(\Gamma_t h_{i,t}^u\right)^{\alpha_u} \tag{2}$$

where α_u and B_t refer to the irregular labor elasticity and a purely transitory technological shock that affects to the irregular production. Note that both, regular and irregular production functions, incorporate labor-augmenting technological process of the form $\Gamma_t = \tau\Gamma_{t-1}$ with $\tau > 1$ which is included in order to ensure a balanced growth path equilibrium in which variables grow at the same rate τ.

As in Orsi et al (2013) and Busato and Chiarini (2004), I assume that goods produced in the underground economy are identical to those produced in the market economy. Therefore, we can normalize prices to 1 and total production of firm i at time t lead as follows:

$$y_{i,t} = y_{i,t}^m + y_{i,t}^u \tag{3}$$

Accordingly to this specification, cost of production consist of capital and regular labor in the case in which firms use the regular production function, and irregular labor in the homologous case. As a result, total cost function is as follows:

$$TC_{i,t} = (1+\tau_t^s)w_t^m h_{i,t}^m + r_t k_{i,t} + w_t^u h_{i,t}^u$$

where τ_t^s is the social security tax rate and r_t is the rental rate of capital from the official production. The cost of labor in the regular market is given by the regular wage, w_t^m, plus a social security contribution per worker. In the irregular production case, labor cost comes from irregular wages, w_t^u, per unit of irregular labor without taxation. Firms are discovered to evade with a probability p_t, and subject in this case to pay the evaded taxes plus a penalty surcharge factor $s > 1$. We can then summarize the profit function as:

$$\begin{aligned} E_t\{\Pi_{i,t}\} &= (1-\tau_t^c)y_{i,t}^m + (1-p_t s\tau_t^c)y_{i,t}^u + \tau_t^c w_t^m h_{i,t}^m \\ &-[(1+\tau_t^s)w_t^m h_{i,t}^m + r_t^m k_{i,t}^m + w_t^u h_{i,t}^u + r_t^u k_{i,t}^u] \end{aligned}$$

Here, I have adopted a different assumption with respect to the model in Orsi et al. (2013). In the original model, irregular labor was deductible in the case of inspection, i.e. a firm that commits tax fraud and is inspected, irregular labor costs can be deducted. This consideration is unrealistic for the Spanish case because the irregular labor is not a legally accounted cost and therefore, it cannot be deduced.

Firms choose $k_{i,t}$, $h_{i,t}^m$ and $h_{i,t}^u$ to maximize expected profits given the production functions. Hence the firm's maximization problem is:

$$\max_{\{h_{i,t}^m, h_{i,t}^u, k_{i,t}\}} E_t\{\Pi_{i,t}\}$$

$$s.t.$$

$$y_{i,t}^m = A_t \left(\Gamma_t h_{i,t}^m\right)^{\alpha} k_{i,t}^{1-\alpha}$$

$$y_{i,t}^u = B_t \left(\Gamma_t h_{i,t}^u\right)^{\alpha_u}$$

The optimal decision for $k_{i,t}$, $h_{i,t}^m$ and $h_{i,t}^u$ are given by the following first order conditions:

$$(1-\alpha)\,\frac{y_{i,t}^m}{k_{i,t}} = \frac{r_t}{1-\tau_t^c} \tag{4}$$

$$\alpha\,\frac{y_{i,t}^m}{h_{i,t}^m} = \frac{w_t^m\,\left(1+\tau_t^s-\tau_t^c\right)}{1-\tau_t^c} \tag{5}$$

$$\alpha_u\,\frac{y_{i,t}^u}{h_{i,t}^u} = \frac{w_t^u}{1-\tau_t^c\,p_t\,s} \tag{6}$$

Since $\tau_t^c \in (0,1)$, the condition that satisfies an interior solution with the existence of underground production in the firm's maximization problem is $(1-\tau_t^c p_t s) > 0$. Otherwise, $(1-\tau_t^c p_t s) \leq 0$ implies $h^{u_t} = 0$ and therefore, no underground production in the economy.

2.2 Representative Household

The representative household has preferences described by the following intertemporal utility function:

$$U_t^h = \sum_{t=0}^{\infty} \beta^t E_0 \{ \frac{((c_t - \gamma_c c_{t-1})/\Gamma_t)^{(1-\sigma)}}{1-\sigma} - B_0 \xi_t^h \frac{(h_t^m + h_t^u)^{1+\xi}}{1+\xi} - B_1 \frac{(h_t^u)^{1+\phi}}{1+\phi} \}$$

where $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution, $\beta \in (0,1)$ is the discount factor, B_0,$B_1 \geq 0$ are parameters controlling for the disutility of working activities, ξ is the inverse labor supply elasticity of aggregate labor supply and ϕ is the inverse labor supply elasticity of underground labor supply. Also, ξ_t^h is a purely transitory labor shock that affects labor supply. The third component of the utility function refers to an additional cost due to work in the underground production sector, it can be interpreted as a greater disutility associated with the lack of social security.

According to this utility specification, households derive utility from a quasi-difference between current consumption and past consumption relative to the rate of technology Γ_t. The habit formation specification implies that an increase in current consumption increases the utility in the current period but decreases utility in the next period. Fuhrer (2000) among others has found that this assumption about household's consumption improves the model solution because it allows to capture into the model the observed gradual hump-shaped response of consumption expenditures to different shocks and particularly, Smets and

Wouters (2007) and Adolfson et al. (2007) have found that the introduction of habit formation improves model fit to the data in bayesian estimation of DSGE models.

Households rent the capital they own according to the following law of motion:

$$k_{t+1} = x_t\, \xi_t^x\, \Psi(x_t/x_{t-1}) + (1-\delta)\ k_t \tag{7}$$

where δ and ξ_t^x are the capital depreciation rate and a purely transitory investment shock that follows an AR(1) process around its steady-sate. Capital adjustment cost are specified as follows:

$$\Psi(x_t/x_{t-1}) = 1 - \frac{\psi}{2}\,(\frac{x_t}{x_{t-1}} - 1)^2$$

Note that this function depends negatively on the investment growth rate and the adjustment cost parameter ψ, i.e. the higher is the investment increase with respect to the previous period the lower is the capital adjustment cost that affects current investment. The implementation of the capital adjustment cost in the law of motion has the final goal of reduce investment volatility and as signalled by Neri (2004), it helps the model in matching some key characteristics of the data.

Like companies, households might evade income taxes by allocating their labor services in the underground production sector. Income derived from production in the regular market will be taxed at the income tax rate τ_t^h. Considering all this information, households' period-by-period budget constraint will be:

$$x_t + c_t = \left(1-\tau_t^h\right)\ (h_t^m\, w_t^m + r_t\, k_t) + h_t^u\, w_t^u \tag{8}$$

The utility maximization problem for the representative household can be described as follows:

$$\max_{\{c_t, k_t^m, h_t^m, h_t^u\}_{t=0}^{\infty}} E_0\{U_t^h\}$$
$$s.t.$$
$$k_{t+1} = x_t\, \xi_t^x\, \Psi(x_t/x_{t-1}) + (1-\delta)\ k_t$$
$$x_t + c_t = \left(1-\tau_t^h\right)\ (h_t^m\, w_t^m + r_t\, k_t) + h_t^u\, w_t^u$$

The solution to this problem must satisfy the following conditions:

$$\lambda_t = (c_t - \gamma_c\, c_{t-1})^{(-\sigma)} - \gamma_c \beta E_t\{\, (c_{t+1} - c_t\, \gamma_c)^{(-\sigma)}\} \tag{9}$$

$$\mu_t = \beta E_t\{\, \left(\lambda_{t+1}\ \left(1-\tau_{t+1}^h\right)\ r_{t+1} + (1-\delta)\ \mu_{t+1}\right)\} \tag{10}$$

$$\begin{aligned} \lambda_t = \xi_t^x\, \mu_t\, &\left(1 - \left(\frac{x_t}{x_{t-1}} - 1\right)^2 \frac{\psi}{2} - \frac{x_t}{x_{t-1}} \left(\frac{x_t}{x_{t-1}} - 1\right)\, \psi\right) \\ &+ \beta E_t\{\ \psi\, \mu_{t+1}\, \xi_{t+1}^x \left(\frac{x_{t+1}}{x_t} - 1\right) \left(\frac{x_{t+1}}{x_t}\right)^2 \} \end{aligned} \tag{11}$$

$$\xi_t^h \, B_0 \, h_t^{\xi} = w_t^m \, \left(1 - \tau_t^h\right) \, \lambda_t \tag{12}$$

$$\xi_t^h \, B_0 \, h_t^{\xi} + B_1 \, h_t^{u\phi} = w_t^u \, \lambda_t \tag{13}$$

where λ_t and μ_t are the Lagrange multipliers for the budget constraint and the law of motion, respectively. Equations (10) and (11) denote the Euler equations which controls for the intertemporal optimality conditions in the households' problem. Equations (12) and (13) provides the optimal schedule for regular and irregular labor.

2.3 Government

The government raises taxes in order to finance public expenditures, g_t. For simplicity, I assume that there is no public debt so that public expenditures are determined on a balanced budget bases, that is:

$$g_t = \tau_t^h (w_t^m h_t^m + r_t k_t) + \tau_t^c \int_0^1 [y_{i,t}^m - w_t^m h_{i,t}^m + p_t s y_{i,t}^u] di + \tau_t^s w_t^m \int_0^1 h_{i,t}^m di$$

Accordingly, fiscal revenues from corporate taxation, household income and social security contributions read as:

$$G_t^c = \tau_t^c \int_0^1 \left(y_{i,t}^m - h_{i,t}^m \, w_t^m + y_{i,t}^u p_t \, s\right) di \tag{14}$$

$$G_t^h = \tau_t^h (\, h_t^m \, w_t^m + r_t \, k_t) \tag{15}$$

$$G_t^s = \tau_t^s \int_0^1 (h_{i,t}^m \, w_t^m) di \tag{16}$$

In addition, total fiscal revenues can be defined as:

$$FR_t = G_t^s + G_t^h + G_t^c \tag{17}$$

Finally, tax evasion in period t can be defined as follows:

$$TE_t = \tau_t^s w_t^u \int_0^1 h_{i,t}^u di + \tau_t^h w_t^u h_t^u + (1 - p_t) \tau_t^c \int_0^1 y_{i,t}^u di$$

2.4 Stochastic Shocks and equilibrium conditions

As mentioned before variables A_t, B_t, τ_t^c, τ_t^h, τ_t^s, p_t, ξ_t^h and ξ_t^x follow an stochastic VAR(1) process of the form:

$$Z_t = (1 - \Phi) Z + \Phi Z_{t-1} + \epsilon_t$$

where $Z_t = (ln(A_t), ln(B_t), ln(\tau_t^c), ln(\tau_t^h), ln(\tau_t^s), ln(p_t), ln(\xi_t^h), ln(\xi_t^x))$, Z contains mean values of the stochastic variables, $\Phi = diag(\rho_a, \rho_b, \rho_c, \rho_h, \rho_s, \rho_p, \rho_{\xi^h}, \rho_{\xi^h})$ and $\epsilon_t = (\epsilon_t^a, \epsilon_t^a, \epsilon_t^b, \epsilon_t^c, \epsilon_t^h, \epsilon_t^s, \epsilon_t^p, \epsilon_t^{\xi^h}, \epsilon_t^{\xi^x})$ is the vector of stochastic shocks which follows a zero mean normal distribution with diagonal variance covariance matrix $\Sigma = diag(\sigma_a^2, \sigma_b^2, \sigma_c^2, \sigma_h^2, \sigma_s^2, \sigma_p^2, \sigma_{\xi^h}^2, \sigma_{\xi^x}^2)$. I consider a symmetric equilibria with a continuum of identical firms that produce the same amount of goods and use the same amount of inputs. Therefore, the market clearing conditions read as follows:

$$c_t + x_t + g_t = \int_0^1 y_{i,t} di$$

$$h_t^u = \int_0^1 h_{i,t}^u di \qquad h_t^m = \int_0^1 h_{i,t}^m di$$

$$k_t = \int_0^1 k_{i,t} di$$

3 Bayesian Estimation of the Model

The model is estimated with Bayesian techniques using mixed frequency data on consumption c_t, investment x_t, regular wages w_t, fiscal revenues from corporate taxation G_t^c, social security contributions G_t^s, fiscal revenues from household income taxation G_t^h, social security contributions tax rate τ_t^s and probability of being inspected by the government p_t for the period 1980:Q1-2015:Q4. I use quarterly frequency for all the variables except for the probability of being inspected and the social security contributions due to their annual definition. Thus, the estimation is based in 144 quarterly observations and 36 annual observations.

In order to link the theoretical model with the real data, I have implemented eight measurement equations according to the following structure:

$$Y_t = \begin{bmatrix} dlCons_t \\ dlInv_t \\ dlWages_t \\ dlFR_t^c \\ dlFR_t^s \\ dlFR_t^h \\ lTax_t^s \\ lProb_t \end{bmatrix} = \begin{bmatrix} \gamma \\ \gamma \\ \gamma \\ \gamma \\ \gamma \\ \gamma \\ ln(\tau_{ss}^s) \\ ln(p_{ss}) \end{bmatrix} + \begin{bmatrix} ln(c_t) - ln(c_{t-1}) \\ ln(x_t) - ln(x_{t-1}) \\ ln(w_t) - ln(w_{t-1}) \\ ln(G_t^c) - ln(G_{t-1}^c) \\ ln(G_t^s) - ln(G_{t-1}^s) \\ ln(G_t^h) - ln(G_{t-1}^h) \\ \hat{\tau}_t^s \\ \hat{p}_t \end{bmatrix} \tag{18}$$

where dl and ln stand for log differences and log, respectively and variables with hat refers to log deviations from their steady-state values.

The model is estimated using a Monte-Carlo Markov-Chain (MCMC) method. Following Smets and Wouters (2003) and Fernandez-Villaverde and Rubio-Ramirez (2001) the estimation procedure can be summarized in three steps. First, I specify prior distributions over a set of structural parameters according to the previous literature and the values of calibrated parameters using Spanish data.

Second, I use Sims (2002) to find the mode of each estimated parameter in the model. For the Metropolis-Hastings (M-H) algorithm we need to start with a high density point, parameters modes are those points with the highest density. Third, I use the M-H algorithm and the Kalman filter with the final goal of finding posterior distributions of the parameters that maximize the likelihood function. The novelty of my estimation comes from the fact that I use mixed frequency data by combining annual and quarterly variables, which allow my to use annual variables without previous transformation. The software used in the estimation is Dynare 4.4.3 launched on Matlab R2015b and the estimation results are based on a Metropolis-Hastings procedure with 4 chains of 300000 replications.

3.1 Data, calibration and prior distributions

Data about consumption, investment, wages and fiscal revenues are provided by the Quarterly Database of the Spanish Economy (BDREMS). All the variables are deflated and expressed in per capita terms using the GDP deflator in 2010 base and dividing by the population aged 15-64 (working age population). The selection of the observable variables clearly depends on the final goal of the estimation. Since I want to estimate the size and trend of the Spanish shadow economy, I have selected those available variables that are the most significant to explain its behavior. In this regard, consumption and investment are used to proxy for total production. Regular wages introduce information about the labor market of the regular production sector.

Also, I have included fiscal revenues from corporate taxation, household income taxation and social security contributions in order to control for the incentives of both, firms and household, to produce in the irregular production sector. In this regard, I include also the evolution of the social security tax rate due to its importance in this model specification.

Finally, the estimation takes into account information about the dynamic of the probability of being inspected by the government p_t. This variable measures risk of evading taxes. Since there is no available data about the real probability of being inspected, I have construct this probability following Busato and Chiarini (2004):

$$p_t = \frac{Inspected\ firms_t}{Total\ number\ of\ firms_t}$$

Data about the number of inspected firms and the total number of firms come from the annual statistical reports of Agencia Tributaria (AETA). Note that this data is only available for the period 1996-2015 and some years are missing.

Bayesian estimation of DSGE models clearly depends on the elicitation of the prior distributions, thus I have chosen priors that are based in previous studies of Smets and Wouters (2003, 2007) for those parameters that are common in these types of model. For the parameters regarding the underground production I have set the priors distribution in line with Orsi et al. (2013). Therefore, the labor elasticity of the regular production α is distributed according to a beta distribution with mean 0.65 and standard deviation 0.02. For the labor elasticity of the underground sector α_u, I have set the same distribution and standard deviation with mean 0.42 in order to reproduce an underground production to output ratio equals to 0.256, which is the estimated average of the Spanish underground economy for the period 1991-2015 (Medina and Schneider (2017)). The capital discount factor δ follows a beta distribution with a mean fixed at 0.010 to ensure a steady-state value of the discount factor $\beta = 0.9927$.

Regarding the parameters controlling for the household's utility, I have chosen gamma prior distributions with mean 1 and standard deviations 0.3 for the inverse of the intertemporal elasticity of substitution σ, the inverse elasticity of total labor supply ξ, and the inverse elasticity of irregular labor supply ϕ. Since this model specification uses habit formation I have set a gamma distribution with mean 0.5 and standard deviation 0.15 for the habit formation parameter γ^c, a value commonly used in the literature (Havranek et al. (2016)). Also, the disutility of working in the underground production sector B_1 follows a gamma prior distribution with mean 50 and standard deviation equals 5. The elicitation of this prior together with the prior distribution of α_u implies $h_u/h = 0.18$ following Hazan (2011). Finally, the parameter controlling for the disutility of total labor (B_0) is updated at any iteration according to the following equation to ensure that at the steady state individuals devote 1/4 of their time to working activities:

$$B_0 = (1 - \tau_{ss}^h) w_{ss}^m \lambda_{ss} h_{ss}^{-\xi}$$

As noted above, the estimation uses data in growth rates with the final goal of using as much information as possible, thus the model needs a parameter controlling for the common quarterly trend growth rate, τ. The distribution of this parameter is normal with mean 1.003 which has been chosen according to the average growth rate of GDP per capita for the period 1980:Q1-2015:Q4. Furthermore, I estimate the steady-sate probability of being inspected p_{ss} and social security tax rate τ_{ss}^s according to beta and gamma distributions with the same standard deviation 0.01 and mean values equals to 0.015 and 0.2973, respectively. The means of both parameters, p_{ss} and τ_{ss}^s, have been set in line with the average value of the probability of being inspected and the social security tax rate for the period 1980-2015.

For the AR(1) processes, there is no much information about the prior distributions of the persistence parameter for the Spanish economy, then I use a beta distribution with a relatively large standard deviation in order to control for a large range of possible values. All the persistence parameter distributions are centered at a mean value of 0.8 and standard deviation 0.1. Finally, for the exogenous processes $\epsilon_t^a, \epsilon_t^a, \epsilon_t^b$, $\epsilon_t^c, \epsilon_t^h, \epsilon_t^s, \epsilon_t^p, \epsilon_t^{\xi^h}$ and $\epsilon_t^{\xi^x}$ the mean values are set in line with the priors used by Orsi et al. (2013).

The rest of the parameters are fixed at a certain value across the whole estimation. More specifically, I have set the penalty that a firm has to pay in case of tax fraud s at 1.875, which is in line with the Spanish law. Also, I have

Table 3: Estimated Parameters

	Density	Prior Mean	Prior Std.	Posterior Mean	95% Confidence Inerval		Posterior Std.
σ_A	'Inv. Gamma'	0.006	Inf	0.010	0.009	0.011	0.001
σ_B	'Inv. Gamma'	0.006	Inf	0.045	0.030	0.060	0.009
σ_c	'Inv. Gamma'	0.006	Inf	0.009	0.008	0.010	0.001
σ_s	'Inv. Gamma'	0.006	Inf	0.017	0.015	0.018	0.001
σ_h	'Inv. Gamma'	0.006	Inf	0.024	0.021	0.026	0.001
σ_p	'Inv. Gamma'	0.006	Inf	0.005	0.001	0.010	0.001
σ_{ξ^h}	'Inv. Gamma'	0.006	Inf	0.035	0.027	0.043	0.003
σ_{ξ^x}	'Inv. Gamma'	0.006	Inf	0.089	0.058	0.119	0.012
α	'Beta'	0.650	0.020	0.704	0.679	0.729	0.015
α_u	'Beta'	0.420	0.020	0.416	0.384	0.448	0.020
δ	'Beta'	0.010	0.001	0.010	0.009	0.012	0.001
ρ_A	'Beta'	0.800	0.100	0.989	0.980	0.999	0.010
ρ_B	'Beta'	0.800	0.100	0.964	0.948	0.980	0.011
ρ_c	'Beta'	0.800	0.100	0.985	0.974	0.997	0.007
ρ_s	'Beta'	0.800	0.100	0.964	0.941	0.988	0.016
ρ_h	'Beta'	0.800	0.100	0.987	0.977	0.997	0.006
ρ_{ξ^x}	'Beta'	0.800	0.100	0.614	0.468	0.761	0.083
ρ_{ξ^h}	'Gamma'	0.800	0.100	0.963	0.939	0.990	0.018
τ^s_{ss}	'Beta'	0.297	0.010	0.299	0.291	0.307	0.004
p_{ss}	'Beta'	0.015	0.001	0.015	0.014	0.017	0.001
σ	'Gamma'	1.000	0.300	1.081	0.778	1.382	0.198
B_1	'Gamma'	50.000	5.000	51.702	43.270	59.779	64.736
ξ	'Gamma'	1.000	0.300	2.059	1.413	2.687	0.276
ϕ	'Gamma'	1.000	0.300	0.500	0.273	0.716	0.222
ψ	'Gamma'	3.000	0.600	3.755	2.629	4.853	0.480
τ	'Normal'	1.003	0.003	1.002	1.001	1.003	0.001
γ_c	'Gamma'	0.500	0.150	0.478	0.368	0.591	0.073

computed the steady-state value for the corporate tax rate and the household income tax rate using the data from the OECD tax database as an average for the period 1980-2015, the resultant values are $\tau^c = 0.3376$ and $\tau^h = 0.19$, respectively.

3.2 Posterior distributions

Table 3 summarizes the prior distributions of the parameters (columns 1-3) along with their estimated posterior distributions. More precisely, it shows the posterior mean along with the 95 percent credible interval and the posterior standard deviation for each estimated parameter (columns 3-6). Also, in figures 1A and 2A in the Appendix we can see a graphical representation of this parameters prior information and their corresponding estimated posterior distributions. Relative to the estimated parameters identification, most parameters seems to be well-identified because the posterior distribution is not centered on the prior mean or it is centered at the prior mean but there is a relatively low dispersion, which means that the prior elicitation is close to the 'real' estimated parameter Forni et al. (2009).

In particular, the regular labor elasticity is estimated somewhat higher than what assumed a priori, which means that regular production is more labor sensitive than what is suggested by the a priori information. For the irregular labor elasticity, the estimated posterior mean implies a slightly reduction of the prior mean to the value 0.4159 with a lower standard deviation. There is no change in the capital depreciation rate and in the probability of being inspected with respect to the prior standard deviation, but in both cases there is an small change in the posterior mean which shows some evidence of identification. The parameter controlling for the steady-state social security tax rate τ^s is strongly identified with a posterior mean equals to 0.2994, as it is shown by the significant reduction in its standard deviation.

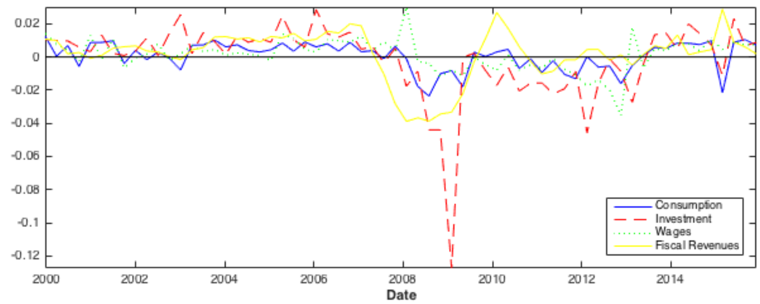
Among AR(1) processes, persistence parameters have been well identified because as in the previous cases the posterior means is significantly different from their a priori mean and vary between 0.96 and 0.99 with the exception of ρ_x, as in the paper by Casares et al. (2016). The same happens for the variance of the exogenous processes, all of them are clearly well identified.

Relating to the parameters affecting the household problem, the inverse of the intertemporal elasticity of substitution σ shows a posterior mean larger than the a priori mean and a lower standard deviation, implying an intertemporal elasticity of substitution lower than 1. For parameter B_1 which is the extra disutility of underground working activities the estimation shows identification problems due to an increase of its standard deviation and a similar posterior mean. The inverse elasticity of aggregate labor supply (ξ)and the inverse elasticity of underground labor supply (ϕ) display a proper identification with clearly different posterior means and lower standard deviations. The estimated mean values for these parameters imply that labor supply in the underground production sector is much more sensitive to changes in the wage rate than labor supply in the regular labor market. This result is consistent with the bayesian estimation of the model implemented by Orsi et al. (2013) for the Italian economy, but the Spanish case shows a larger difference between this two parameter, suggesting that the underground production sector in Spain is more sensitive than the its Italian counterpart to movements in the wage rate. For the habit formation parameter γ^c, I found a posterior mean equals 0.4776 which is relatively similar to the prior mean and it is in line with other estimations (Havranek (2016). Finally, parameters controlling for the labor augmenting technological progress (τ) and the capital adjustment cost (ϕ) seem to be also well identified.

4 Robustness Check

In this section I elaborate a simple test to check the robustness of the estimated results presented above. More precisely, I estimate the model with a different length of data to test whether the estimated results change significantly. The main reason to conduct such a robustness test is given by the huge increase in volatility that occurs during the financial crisis that started at the end of 2007. From the last quarter of 2007 there is an important increase in the variability of the series that I use in the bayesian estimation. Since I am assuming constant

Figure 1: Growth Rates



variances in the bayesian estimation and in the Kalman filter, this increase in the variance of the data can reduce estimates reliability as is shown by McConell and Perez-Quirs (1998). Therefore, I estimate the model for the period 1980:Q1-2007:Q3 in order to study the effects of the financial crisis in the estimation results.

The estimation was carried out with the same priors distributions of the parameters and the same scale parameter. For the mode computation I used Sims (2002) algorithm as in the previous estimation and results are based in a Metropolis-Hastings algorithm with 4 chains of 300000 replications. Table 4 presents posterior means and posterior standard deviations of both, benchmark estimation and the estimation based on the sub-period 1980:Q1-2007:Q3.

Overall, results confirm the benchmark findings in terms of both, posterior means and posterior standard deviations. More precisely, variances of the exogenous processes are larger for the estimation based in the whole period, which is pretty obvious if we take into account the variation in the data, but the differences do not appear to be important for the model estimation since the estimated dynamic of the underground production does not change significantly. The labor elasticities of regular and irregular production and the capital depreciation rate are almost the same and this happens also with the posterior standard deviations, which shows a robust estimation of these parameters. For the persistence parameters of the exogenous processes I did not find a considerable difference between the posterior means except for ρ_{ξ^x}. This result is clearly due to the huge increase in the variation of investment growth rates, which in 2009 reach quarterly growth rates of more than -12%, as it is shown in figure 1. The most appreciable difference occurs in the case of the household utility parameters, whose estimates turn out to be significantly different with respect to the benchmark specification. For the inverse elasticity of substitution there is an important difference, which shows that, using the new length of data, consumption is less sensitive to movements in the real interest rate with respect to the previous estimation. These increase of the inverse elasticity of substitution σ could be due to the increase in consumption variability in the aftermath of

Table 4: Robustness Check

	Posterior Mean		Posterior Standard Deviation	
	Benchmark Model	1980:Q1-2007:Q3	Benchmark Model	1980:Q1-2007:Q3
σ_A	0.010	0.009	0.001	0.001
σ_B	0.045	0.030	0.009	0.009
σ_c	0.009	0.010	0.001	0.001
σ_s	0.017	0.012	0.001	0.001
σ_h	0.024	0.023	0.001	0.001
σ_p	0.005	0.005	0.001	0.001
σ_{ξ^h}	0.035	0.028	0.003	0.005
σ_{ξ^x}	0.089	0.079	0.012	0.018
α	0.704	0.695	0.015	0.015
α_u	0.416	0.420	0.020	0.020
δ	0.010	0.010	0.001	0.001
ρ_A	0.989	0.971	0.010	0.004
ρ_B	0.964	0.957	0.011	0.010
ρ_c	0.985	0.990	0.007	0.007
ρ_s	0.964	0.961	0.016	0.015
ρ_h	0.987	0.991	0.006	0.006
ρ_{ξ^x}	0.614	0.686	0.083	0.087
ρ_{ξ^h}	0.963	0.962	0.018	0.018
τ^s_{ss}	0.299	0.300	0.004	0.005
p_{ss}	0.015	0.015	0.001	0.001
σ	1.081	1.253	0.198	0.208
B_1	51.702	50.630	6.474	5.020
ξ	2.059	1.663	0.276	0.383
ϕ	0.500	0.716	0.222	0.136
ψ	3.755	3.338	0.480	0.665
τ	1.002	1.005	0.001	0.001
γ_c	0.478	0.371	0.073	0.076

the crisis. Also, results changes for the regular labor supply elasticity and the irregular labor supply elasticity suggesting that the sensitivity of both, regular and irregular labor, to changes in the wage rate it is closer than in the benchmark estimation. However, these results have very similar implications for the model and the dynamic of the main variables remains almost equal. Finally, for the cost of supplying labor to the irregular production and the parameter driving the capital adjustment cost I found very similar estimates.

Overall, this robustness check shows a pretty good estimation of the DSGE model as it is corroborated by most of the posterior means and standard deviations of the parameters.

5 Dynamics of the Underground Economy in Spain

In this section I present estimates of the underground economy in Spain over the period 1980:Q1-2015:Q4, with the aim of understanding the main driving forces behind the predicted dynamics. Also, this section presents a description

Figure 2: Underground to Output Ratio

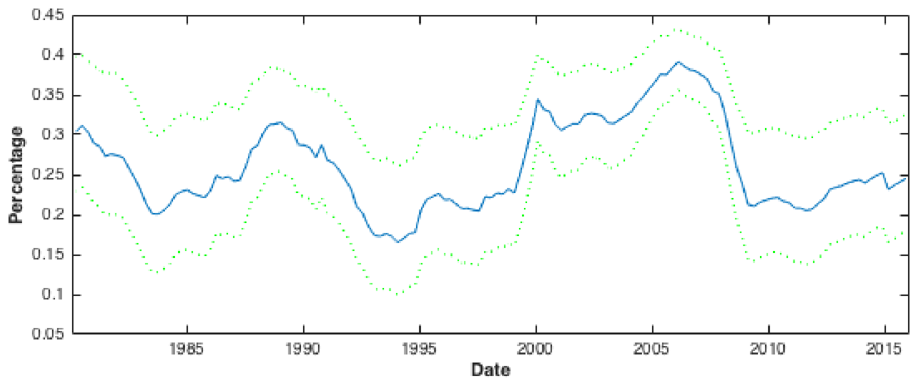


Figure 2: Smoothed estimates of the Underground to output ratio (blue line) together with the 95% confidence interval (green dashed lines).

of the main business cycle statistics with the final goal of understanding the behavior of the estimated model.

Figure 2 shows the smoothed estimates of the irregular production as a fraction of total output along with the 95 percent confidence interval for the estimated period. I found a large variability in the size of the Spanish shadow economy, ranging from to a minimum of 16% to a maximum of about 35%. In particular, the model predicts that between 1980-1989 there is a complete cycle in which the underground economy starts at a 30% of the total production, then it decreases up to the 20% level in 1984 and comes back to the starting point in the following five years. Second, an important decrease occurs between the years 1990 and 1994 leaving the underground economy at the lowest level (16%) in the analyzed years. From this point there is a long process of growth that lasts up to 2007, a total of 13 years, which situates its level of importance in more than 35% over the total production, and it is followed by a significant reduction that ends in the last quarter of 2008. Finally, notice that there is a considerable increase in the irregular production to output ratio that occurs during the financial crisis (2007). As a matter of fact, after a substantial decrease at the beginning of the crisis, the size of the underground has substantially increased (about 5 percentage points) in all the subsequential periods. This result is consistent with Gestha (2014), which estimates an important increase of the underground economy since 2008.

In order to explain the dynamics of the underground economy, I next study the factors that affects its variation. To this end, I present the historical shock decomposition of the Spanish shadow economy for the considered period in percentage deviations from the steady-state (Figure 3), to disentangle the relative contribution of shocks to the dynamic of the estimated underground production. This variations are determined by the estimated stochastic innovations

Figure 3: Historical Decomposition

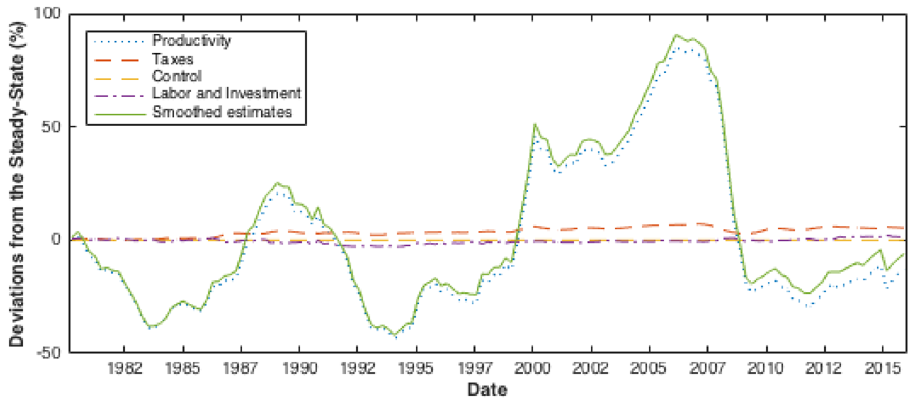


Figure 2: Historical decomposition of the underground production. The productivity component includes productivity shock of both regular and irregular sector. The taxes factor includes shocks regarding social security contribution tax rate, corporate tax rate and household income tax rate. Labor and Investment component refers to labor and investment shocks while the control component includes the probability of being inspected.

that are specified in the model, which in the picture have been summarized into productivity, labor and investment, taxes and control components. The first one refers to the changes in the shadow economy that comes from changes in productivity of both regular and irregular sector (A_t and B_t) and, as the picture illustrates, is the most important factor in explaining the variations in the underground production. Most likely, this result is driven by the positive consumption-underground production relationship. The intuition is straightforward. When the economy experiences a boost in productivity, consumption and output both increase and therefore firms uses more irregular labor to produce in order to avoid payment of taxes. Conversely, since firms have to pay corporate tax profits net of labor costs and irregular labor is not deductible, it is very likely that firms find unprofitable to produce underground output in recession, thus explaining why I found an important decrease in the size of the underground economy when there is a significant contraction in consumption.

This interpretation is supported by the orthogonalized impulse response functions to a shocks in both, regular and irregular productivity. On the one hand, Figure 10 in the Appendix shows that a positive shock in regular productivity produces a contraction in the irregular production and therefore, in the ratio of underground economy and tax evasion. On the other hand, a positive shock in the underground productivity originates an increase in total production and irregular production, while it generates a negative response of regular output, as it is shown in Figure 11. Relating to this, the decrease in the underground economy ratio in case of a regular productivity shock is driven by two negative effects, the total output increase and the irregular production decrease, while the increase in the underground ratio due to a irregular productivity shock

Figure 4: Underground Production & Taxes

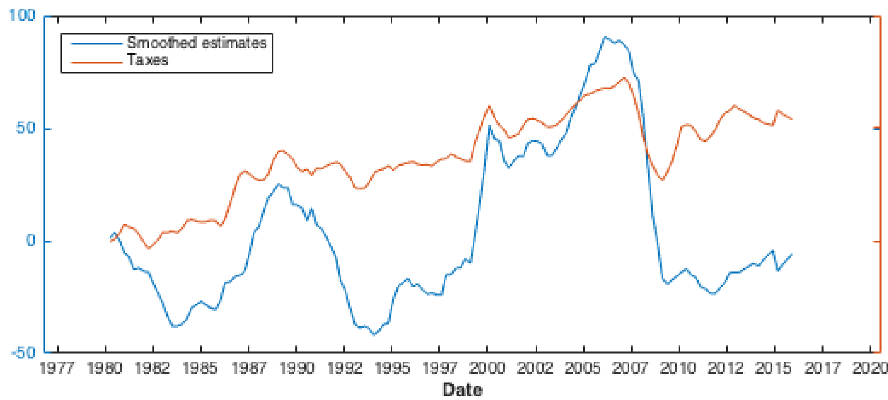


Figure 4: Smoothed estimates of underground production and Taxes component. Note that the two series are plotted in different axes.

depends on both total output negative effect and irregular production positive effect. This result is due to the composition of the underground economy ratio, which depends positively on y^u and negatively on y. Thus, since in case of a regular productivity shock there is a rise in y and a reduction in y^u the underground ratio decreases by more than the reduction in the irregular production, whereas the increase in this ratio is lower than the rise in underground output when there is an irregular productivity shock.

Coming back to the historical decomposition of the underground production, labor and investment shocks and the probability of being inspected do not explain a significant portion of the variation in the irregular production. Therefore, the remaining differences between the variance of the productivity shocks and the smoothed estimate of the irregular production can be explained by the variation in taxes. Regarding to this, Figure 4 depicts the evolution of the tax factor together with the underground production, showing that the two series are in general negatively correlated. To explain these findings, in Figures 12-14 (see the appendix) I report the response of the economy to a temporary increase in tax rates. As the pictures illustrate, an increase in each tax rate produces a contraction in total production y_t, regular production y_t^m, consumption c_t and investment x_t among others, while it produces a rise in irregular labor h_t^u, and therefore in the underground production y_t^u. Therefore, since the ratio of underground economy (y_t^u/y_t) depends positively on irregular production and negatively on total production, it increases by more than the expansion in the irregular economy. This is the so called resource reallocation effect, which has been emphasized in previous studies by Orsi et al. (2013) and Basile et al. (2012) among others. More precisely and since the model specification takes into account labor as the unique input in the underground production, this could be denominated labor reallocation effect, which in this case transfers labor from the regular to the irregular production. Notice that this reallocation effect is larger when the increase in tax rate affects personal income (shock on τ^h) than with a shock in τ^c or τ^s, which is due to the change in the individual preferences that are more affected by the household income tax rate. Also, it is important to take into account the variation in tax evasion (TE) when there is a shock in any of the taxes. As we can see in the orthogonalized impulse response function to shocks in any of the taxes, there is an expansion in tax evasion for all the cases, which is consistent with the results in Orsi et al. (2013) and Jung and Trandel (1994) among others, and highlights the importance of taking into account the existence of tax evasion when choosing proper fiscal policies. In Section 6 I implement a fiscal policy analysis with the final goal of emphasizing the importance of this issue.

Concerning the dynamic of the economy with the existence of irregular sector, it is important to analyze the main business cycle statistics in order to understand the mechanism that describe its behavior. In table 6, I present standard deviations, correlation coefficients with respect to irregular output (y^u) and autocorrelation coefficients for each variable. Two main results are worth emphasizing. First, as noted before irregular output turns is much more volatile than regular output, being the standard deviation of the former about 4 time higher than the standard deviation of regular output. This finding mirrors volatilities in labor variables, suggesting that regular labor is less sensitive than the irregular one to the business cycle.These results also implies that the regular labor market is less sensitive than the irregular labor market. Second, the estimated results provide evidence in favor of a double business cycle in the Spanish economy, being regular and irregular production negatively correlated. The countercyclical nature of the underground economy is also supported by the negative correlation with regular inputs and fiscal revenues, as those variables move instead procyclically.

6 Policy Implications

This section conducts a fiscal policy analysis, in which I evaluate the steady-state Laffer curve for corporate tax rate and household income tax rate in an economy with and without underground sector. The model specification in the case in which there is no irregular sector is similar to the one explained in section 2, and I use the same estimated parameters for both economies. This analysis is carried out with the final goal of highlighting the importance of taking into account the importance of the underground economy in the implementation of fiscal policies. In this section I give an special importance to the effects of those fiscal policies over social security contributions due to its importance in social welfare.

Table 5: Business Cycle Statistics

	Standard Deviation	Correlation (y^u) %	Autocorrelation (1) %
y	0.0195	69.89	80.60
y^m	0.0199	-39.43	77.50
y^u	0.0821	100.00	72.83
h^m	0.0212	-46.69	74.69
h^u	0.0587	95.83	73.21
h	0.0178	-23.16	76.68
k	0.0063	0.26	95.03
w^m	0.0141	13.90	68.84
w^u	0.0308	83.77	63.63
c	0.0181	32.50	81.26
x	0.0837	16.11	88.29
G^h	0.0365	-19.30	72.62
G^s	0.0215	-36.05	76.40
G^c	0.0302	-18.12	73.72
FR	0.0224	-30.65	75.04
TE	0.0834	98.79	72.69
$Ratio$	0.0698	97.98	71.83

Table 5: This table presents estimates for standard deviation, correlation with irregular production and first order autocorrelation for the main variables in the model. Estimated results are based on a stochastic simulation with the parameters fixed at their posterior mean values and using Hp-filtered variables.

6.1 Corporate Tax Rate

Figure 5 presents the corporate tax rate steady-state Laffer curve for both, the benchmark model and the model without underground production. The curves take into account total fiscal revenues for each value of τ^c keeping all the remaining parameters fixed to their posterior mean values. The vertical straight line depicts the steady-state value of corporate tax rate computed as the average tax rate for the period 1980-2015. Also, I present the response of fiscal revenues from social security contributions and a sensitivity analysis in which there is an increase in the utility cost of supplying labor into the underground sector (B_1).

First, the estimated model predicts that Spain is in the left hand side of the optimal corporate tax rate for both model specification, the benchmark and the model without irregular production. [4] It shows that it is possible to increase total fiscal revenues by rising corporate tax rate. However, I found that the increase in corporate tax rate has negatives effects on fiscal revenues that comes from social security contributions, which generates a decrease in social security contributions that are used to finance social benefits. More precisely,

[4]This result is similar to the estimated Laffer curves in Trabandt and Uhlig (2009) and Busato and Chiarini (2013) for Italy and US, respectively.

Figure 5: Steady-State Laffer Curve (Corporate Taxation)

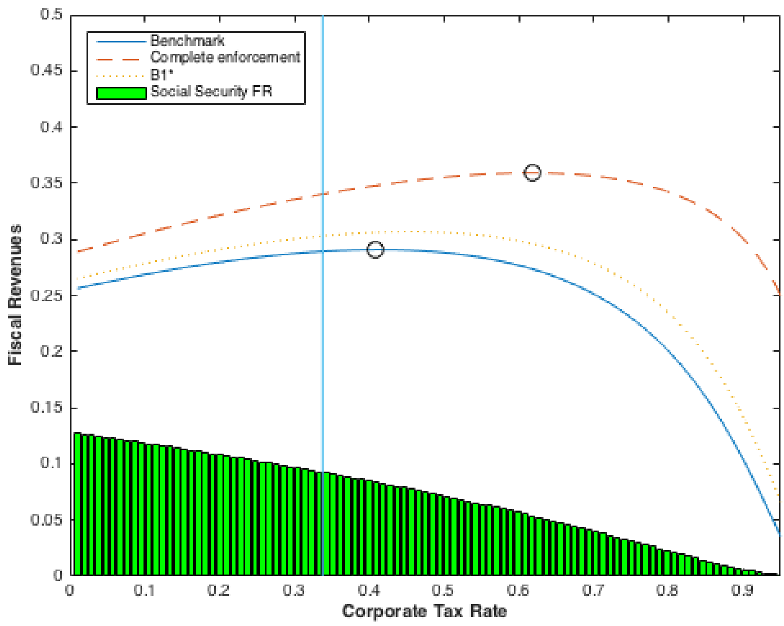


Figure 5: This picture shows the steady-state Laffer curve for the benchmark case, the complete enforcement case and fixing B_1 at twice the estimated value as a function of corporate tax rate τ^c. Green bars refers to social security fiscal revenues. All the other parameters are kept fixed to their posterior mean values.

this negative effect produces an important increase in social security tax evasion as can be observed in figure 6, which is primarily due to the expansion in underground production that occurs with higher corporate tax rate.

The mechanism that drives this effect is illustrated in the orthogonalized impulse response function to a shock in τ^c (figure 12 in the appendix). A positive shock in corporate tax rate produces an increase in corporate fiscal revenues and due to the labor reallocation effect there is an important reduction in regular labor, which goes to the irregular production. However, as we can see the reduction in regular labor is lower than the increase in underground labor. This happens because, since firms pay corporate taxes net of labor costs the rise in corporate tax rate produces an increase in the relative price of producing using capital. Therefore, the expansion of the irregular production is due to two effects: 1) labor reallocation effect and 2) capital effect, firms produce more using labor instead of capital. The irregular production definition which uses labor as the unique input, reproduce this result because in contrast to the original model in Orsi et al. (2013) there is no capital counterpart in the irregular sector that firms can use in order to avoid corporate taxes.

Notice that the optimal corporate tax rate that maximize fiscal revenues in the full enforcement economy would instead lead the economy with underground sector to the slippery slope side of the Laffer curve, resulting therefore in a decline of fiscal revenues. This finding clearly highlights the importance of taking into account the underground sector and tax evasion in the fiscal policy design. The estimated increase in fiscal revenues due to a rise in the tax rate is also larger than in the previous case, which means that is more likely that the government rise taxes. Furthermore, the effects of a variation in total fiscal revenues must be taken into account, but also the reaction of fiscal revenues and tax evasion that comes from other taxes. In this case the marginal increase of rising the corporate tax rate is very low and the negative effects over social security fiscal revenues and its tax evasion must be considered.

The parameter B_1 refers to the extra cost that the individual has to assume in order to supply labor in the irregular production sector. Although in the model this parameter is a primitive parameter reflecting the household willingness to work in the underground sector, it can be more in general interpreted as controlling for those elements that are left unspecified in the model such as the quality of institutions, culture elements (tax morality), labor market policy and enforcement legislation. All these elements might be affected by the government through properly oriented policies. An example of these type of interventions are policies on social benefits, which might affect the willingness of household to work in the underground sector. In this respect, we then conduct a sensitivity analysis by testing how the predictions of the model change by setting parameter B1 twice as larger than its estimated posterior mean. Results are provided in Figure 5 for the estimated Laffer curve and in Figure 6, which displays the effect of a higher B1 to the evasion in social security contributions. As the picture illustrate, at actual tax rate there is a significant improve in fiscal revenues associated with a decline in tax evasion from social security contributions. This is a very interesting result because it means that the government can increase fiscal revenues reducing tax evasion without negative effects over other taxes. In addition, optimal fiscal revenues can be achieved by implementing positive fiscal policies that improves household welfare. In this regard, it is possible to achieve optimal fiscal revenues by rising the disutility of working in the irregular sector B_1 from 51.7 to 53, without negative effects on social security contributions and tax evasion.

6.2 Household Income Tax Rate

As in the previous subsection, figure 7 presents the estimated steady-state Laffer curve for the benchmark model, the model without underground sector and the steady-state Laffer curve with a new value of B_1. Also, as before it depicts fiscal revenues that comes from social security tax rate with the final goal of analyzing the effects of a variation in household income tax rate. The vertical straight line represents the steady-state tax rate value.

It is obvious that fiscal revenues are always lower in the benchmark economy than in the economy without tax evasion because agents can avoid income

Figure 6: Social Security Tax Evasion

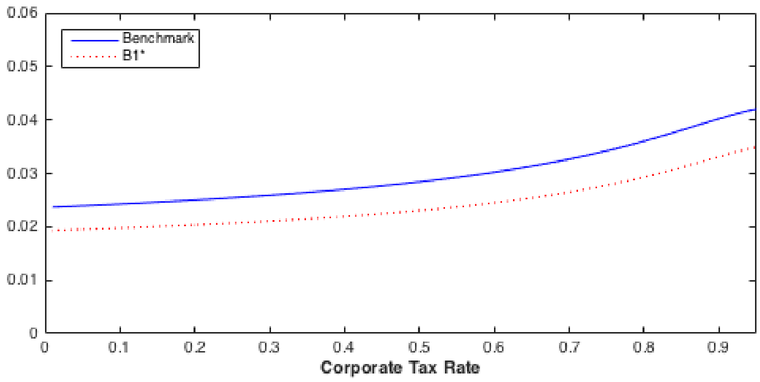


Figure 6: This figure shows the level of tax evasion for both the benchmark case and the case with B_1^* as a function of τ^c.

taxes by reallocating capital and labor from the regular to the irregular sector. Results in figure 7 shows that there is a clear difference between the optimal tax rate that maximizes fiscal revenues in the benchmark model and the model without underground economy. This difference is even larger than in the case of corporate tax rate, which may results in greater efficiency looses.

As a simple example I consider a scenario in which, taking an economy with irregular sector, the government sets the household income tax rate at the complete enforcement optimal level $\tau^h = 0.59$. According to the model estimation, total fiscal revenues would be 18 percentage points lower than the optimal fiscal revenues in the benchmark economy. This result exhibits the relevance of the information that appears when there is tax evasion in selecting suitable fiscal policies.

As in the case of corporate tax rate, an important reduction in social security fiscal revenues occurs when there is a rise in the income tax rate, showing that taxes affects the individual willingness of working in the underground production sector. My findings suggest that it is possible to increase total fiscal revenues by rising household income tax rate from 0.19 to 0.34, resulting in a quantitatively important positive change of almost 5 percentage points more. However, this rise in taxes affects negatively to the social security budget by reducing its fiscal revenues by 14 percentage points.

Finally, the sensitivity analysis over B_1, in which the cost of suppling labor to the irregular market is twice the original estimated value, shows that it is possible to reach the optimal quantity of fiscal revenues. More precisely, by doubling the value of B_1 fiscal revenues increases in the same quantity than in the case of optimal taxation, without negative effects in social security contributions and tax evasion. Therefore, this result suggest that increasing the cost of supplying labor to the irregular production sector is pareto optimal in terms

Figure 7: Steady-State Laffer Curve (Household Income)

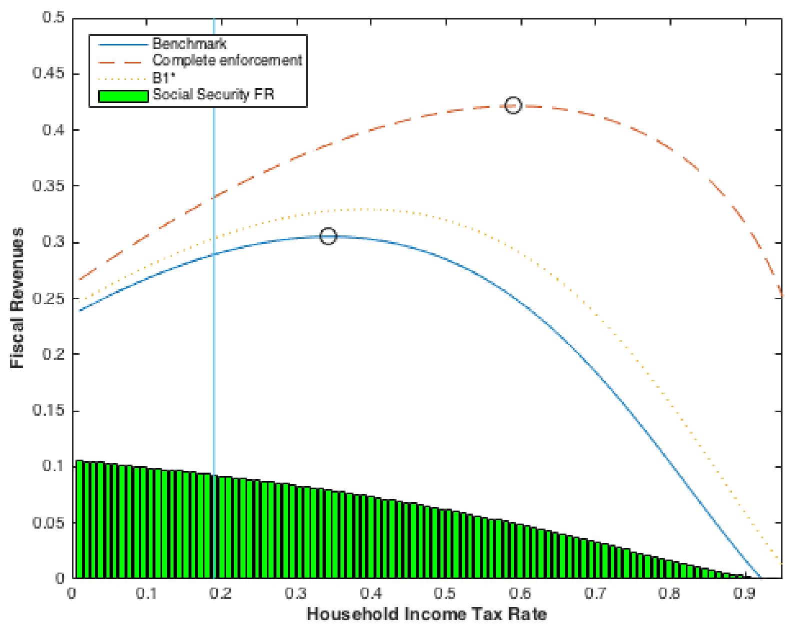


Figure 7: This picture shows the steady-state Laffer curve for the benchmark case, the complete enforcement case and fixing B_1 at twice the estimated value as a function of corporate tax rate τ^h. Green bars refers to social security fiscal revenues. All the other parameters are kept fixed to their posterior mean values.

of fiscal revenues, while increasing household income tax rate affects negatively to social security contributions and may decrease social welfare.

7 Conclusion

In this paper I have presented an estimation of the Spanish underground economy along with the main variables that explain its variation for the period 1980:Q1-2015:Q4. I found that its size has a widely variance ranging between 16% and more than 35% of the total output, due to its dependence on the labor market. Since the irregular production uses labor as the unique input in the production process, it cannot use capital as an alternative to unproductive labor, i.e. whenever labor is unproductive the unique way to avoid this unproductiveness is to increase regular production. This result could explain the substantial increase after the beginning of the financial crisis because the fall in wages produces a higher labor productivity, which means that firms have incentives to avoid corporate taxes by hiring more irregular labor.

Regarding to the fiscal policy analysis, I found that the Spanish government can increase fiscal revenues by rising both corporate and household income tax rate. However, as noted above increasing taxes generates negative effects on social security contributions, which may damage social welfare. Therefore, it is important to take into account these negative effects in order to achieve the optimal fiscal policy. On this matter, I present a sensitivity analysis over the cost of supplying labor to the irregular sector B_1 in which this parameter doubles its value showing a significant increase on fiscal revenues. As noted above, this rise in the cost parameter could be due to different policies but also it is possible to achieve the same result by reducing the cost of supplying total labor B_0 relative to B_1. This means that the government can reduce the size of the irregular production sector, and therefore tax evasion by implementing policies that affects positively the household utility and without negative effects on social security contributions.

These results shows another perspective for reducing the size of the underground economy, and presents new research topics focused in compute the effects of those policies in the size of the informal sector.

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7.1 Appendix

Figure 8: Prior and Posterior Distributions (1)

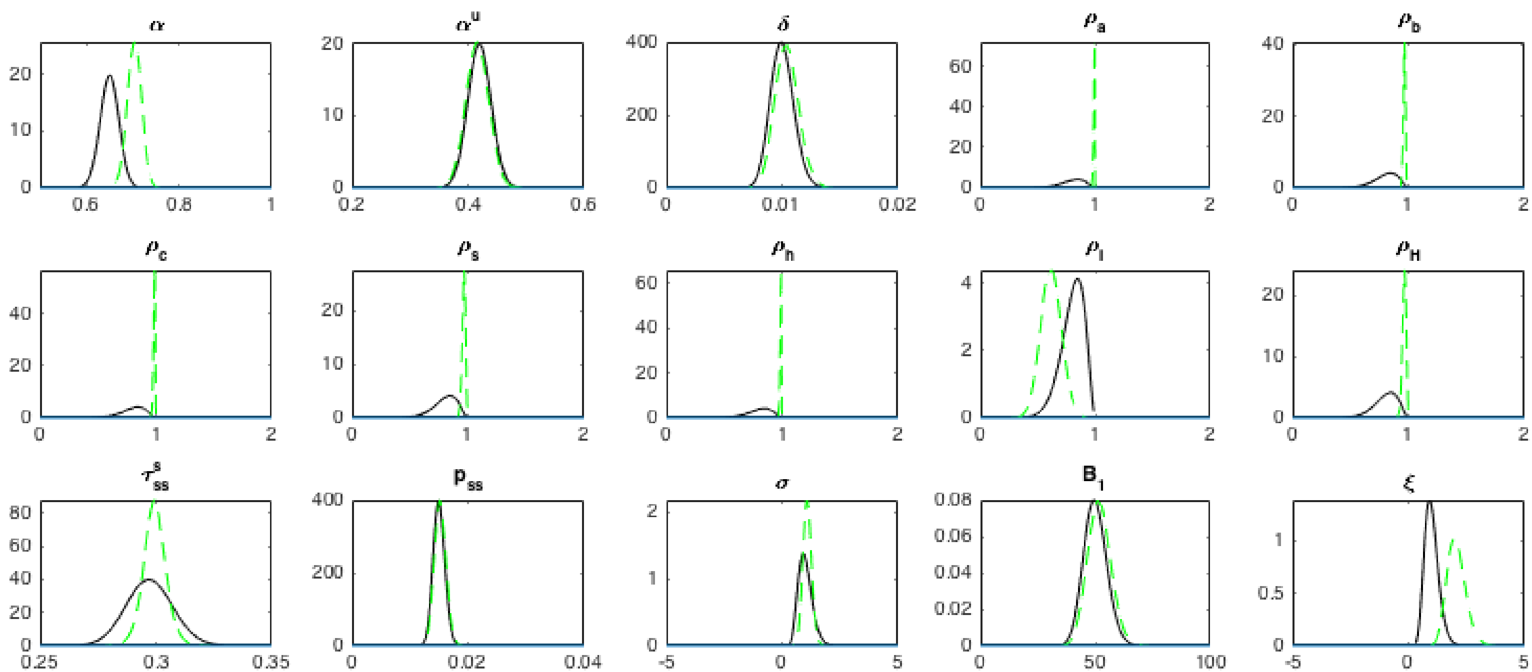

Figure 9: Prior and Posterior Distributions (2)

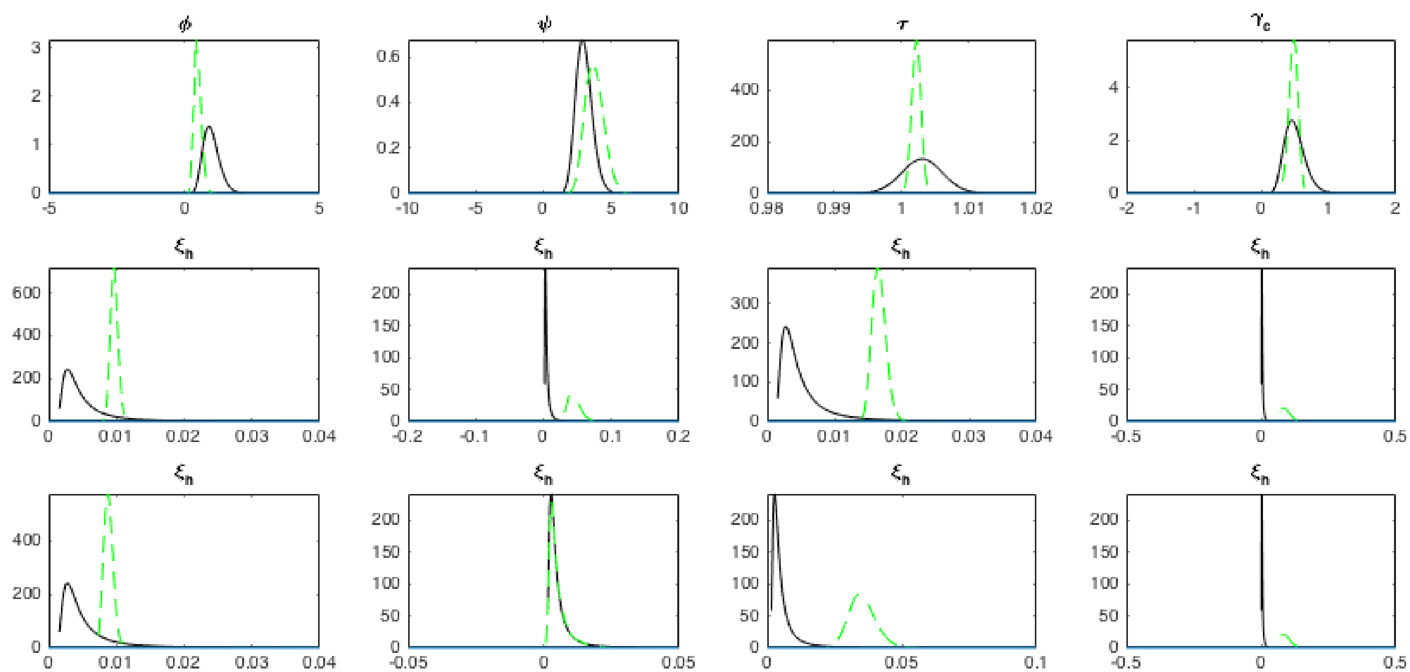

Figure 10: Orthogonalized Impulse Response Function to a shock in A_t

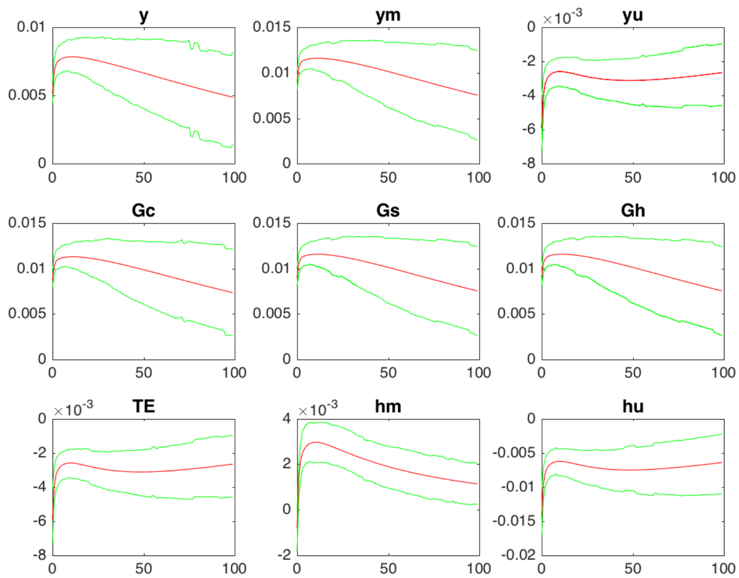

Figure 11: Orthogonalized Impulse Response Function to a shock in B_t

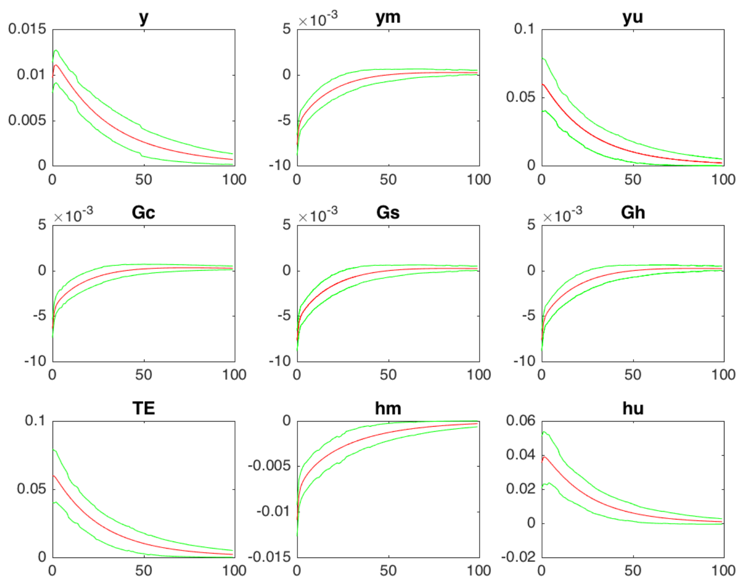

Figure 12: Orthogonalized Impulse Response Function to a shock in τ_c

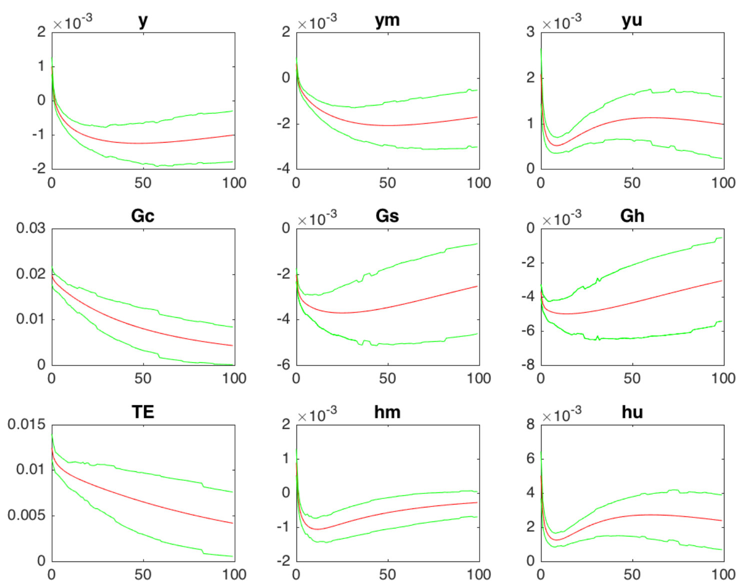

Figure 13: Orthogonalized Impulse Response Function to a shock in τ_s

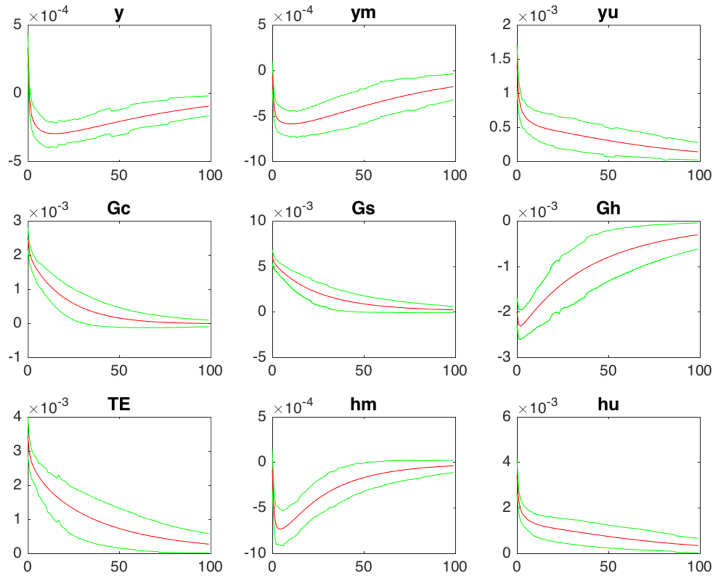

Figure 14: Orthogonalized Impulse Response Function to a shock in τ_h